The Design of Directional Couplers Using Exponential Lines in Inhomogeneous Media

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Abstract—Coupled exponential transmission lines in inhomogeneous media are analyzed and an accurate solution is obtained. The solution is used to design microwave directional couplers. The designed couplers have much improved coupling characteristics compared with couplers using uniform lines. Practical measurements show very good agreement with theoretical results.

I. Introduction

NONUNIFORM transmission lines (NUL) have been analyzed and their applications reported by a number of authors [1]–[10]. An exact analysis is only possible for a few cases, for example, exponential and Bessel transmission lines [6]. Coupled nonuniform transmission lines (CNUTL) offer additional design parameters that could be adjusted to achieve the desired performance. When used as directional couplers CNUTL offer much wider bandwidths than can be obtained by coupler uniform transmission lines (CUTL). The solutions of CNUTL reported so far do not apply to microstrip networks where the medium is inhomogeneous and the modes of propagation have different phase velocities. In this paper a solution for the modes of propagation of coupled exponential transmission lines (CETL) in inhomogeneous media is derived and the design of directional couplers using these lines is described. Measured results on an implemented circuit are reported.

II. The Analysis of CETL in Inhomogeneous Media

Consider the CETL as illustrated in Fig. 1(a). The first-order differential equations representing CETL are

$$\frac{\partial v}{\partial x} = -L\frac{\partial i}{\partial t} \tag{1a}$$

$$\frac{\partial i}{\partial x} = -C\frac{\partial v}{\partial t} \tag{1b}$$

where

$$v = \begin{bmatrix} v_{\mathrm{I}} \\ v_{\mathrm{II}} \end{bmatrix} \text{ and } i = \begin{bmatrix} i_{\mathrm{I}} \\ i_{\mathrm{II}} \end{bmatrix}$$

v_{I} and i_{I} the instantaneous voltage and current on line I at x, respectively,

v_{II} and i_{II} the instantaneous voltage and current on line II at x, respectively,

$$L = \begin{bmatrix} L_{11}(x) & L_{12}(x) \\ L_{21}(x) & L_{22}(x) \end{bmatrix}$$

$$C = \begin{bmatrix} C_{11}(x) & C_{12}(x) \\ C_{21}(x) & C_{22}(x) \end{bmatrix}$$

in which L, C are the inductance and capacitance (per unit length) matrices, both are positive definite. The diagonal elements in L and C represent the self inductances and capacitances and the off-diagonal elements represent the mutual inductances and capacitances. For two identical coupled lines the two diagonal elements are equal and the two off-diagonal elements are also equal.

Due to the reflection symmetry, the behavior of CETL can be completely described by the superposition of the two fundamental even and odd modes as shown in Fig. 1(b). Throughout this paper, subscripts e and o, refer to the even and odd modes, respectively.

The voltages $v_{e,o}$ and currents $i_{e,o}$ of the even and odd modes are related to the total voltages and currents on the lines by

$$v_{e,o} = \tfrac{1}{2}(v_{\mathrm{I}} \pm v_{\mathrm{II}}) \tag{2a}$$

and

$$i_{e,o} = \tfrac{1}{2}(i_{\mathrm{I}} \pm i_{\mathrm{II}}). \tag{2b}$$

The differential equations relating $v_{e,o}$ and $i_{e,o}$ are then obtained from (1) and (2) and are given by

$$\frac{\partial v}{\partial x}e,o = -L_{e,o}(x)\frac{\partial i}{\partial t}e,o \tag{3a}$$

$$\frac{\partial i}{\partial x}e,o = -C_{e,o}(x)\frac{\partial v}{\partial t}e,o \tag{3b}$$

where

$$L_{e,o}(x) = L_{11}(x) \pm L_{12}(x)$$

$$C_{e,o}(x) = C_{11}(x) \pm C_{12}(x)$$

the plus and minus signs are for the even and odd modes, respectively.

For a CETL the inductances $L_{e,o}(x)$ and capacitances $C_{e,o}(x)$ vary with distance x according to

$$L_{e,o}(x) = L_{e,o}e^{\alpha_{e,o}x}$$

$$C_{e,o}(x) = C_{e,o}e^{-\alpha_{e,o}x}.$$

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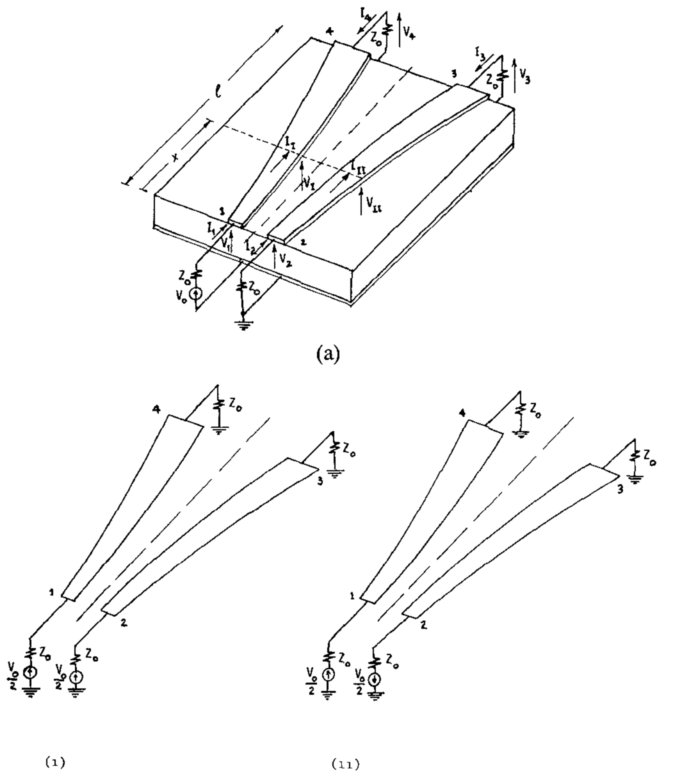

(a)

(i) (ii)

(b)

Fig. 1(a). A microstrip CETL. (b) Excitation of the fundamental modes by means of constant voltage generators; (i) even mode, (ii) odd mode.

The constants α_e and α_o depend on the exponential variation of the width of the lines and the gap between them and are related by conditions of physical realizability as discussed later.

The steady-state solutions for the sinusoidal excitation $e^{-j\omega t}$ will be similar to that of single ETL for each mode

$$V_{e,o}(x)=V_{1e,1o}^{+}e^{\alpha_{e,o}x/2}e^{-j\beta_{e,o}x}+V_{1e,1o}^{-}e^{\alpha_{e,o}x/2}e^{j\beta_{e,o}x} \tag{4}$$

$$I_{e,o}(x)=\frac{V_{1e,1o}^{+}}{Z_{e,o}(x)}e^{\alpha_{e,o}x/2}e^{-j\beta_{e,o}x}-\frac{V_{1e,1o}^{-}}{Z_{e,o}^{*}(x)}e^{\alpha_{e,o}x/2}e^{j\beta_{e,o}x} \tag{5}$$

where

$\beta_{e,o}=\sqrt{\beta_{oe,oe}^{2}-(\alpha_{e,o}/2)^{2}}$	even (odd) phase constant,
$\beta_{oe,oo}=\omega\sqrt{L_{e,o}C_{e,o}}$	even (odd) phase constant of the corresponding CUTL,
$Z_{e,o}(x)=Z_{oe,oo}e^{\alpha_{e,o}x}$	even (odd) characteristic impedance at x,
$Z_{oe,oo}=\sqrt{\frac{L_{e,o}}{C_{e,o}}}\,e^{-j\theta_{e,o}}$	even (odd) characteristic impedance at $x=0$,
$\theta_{e,o}$	$=\tan^{-1}(-\alpha_{e,o}/2\beta_{e,o})$.

The voltage and current on each line are given by

$$V_{\mathrm{I,II}}=V_e\pm V_o$$

and

$$I_{\mathrm{I,II}}=I_e\pm I_o.$$

III. Coupled Exponential Transmission Line Directional Couplers

The behavior of a coupler is best described by the use of the scattering parameters.

The four-port scattering matrix can be written in the form

$$S=\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14}\\ S_{21} & S_{22} & S_{23} & S_{24}\\ S_{31} & S_{32} & S_{33} & S_{34}\\ S_{41} & S_{42} & S_{43} & S_{44}\end{bmatrix}. \tag{6}$$

Due to the symmetry of the CETL, we have

$$S_{11}=S_{22} \quad S_{21}=S_{12} \quad S_{31}=S_{24} \quad S_{41}=S_{32}$$

and

$$S_{33}=S_{44} \quad S_{13}=S_{24} \quad S_{23}=S_{14} \quad S_{43}=S_{34}.$$

The independent scattering parameters are chosen to be S_{11}, S_{21}, S_{31}, S_{41}, S_{13}, S_{23}, S_{33}, and S_{43}. Each line of the CETL can be treated, for each mode of excitation, as a two-port network for which all the equations of the single ETL are valid. Using the superposition principle, the scattering parameters S_{11}, S_{21}, S_{31}, and S_{41}, normalized to the impedance Z_o, are found to be

$$S_{11,21}=\frac{1}{2}\frac{1}{1+Z_o/Z_{oe}}\cdot\frac{(1-Z_o/Z_{oe})e^{j2\beta_e l}+\rho_{2e}(1+Z_o/Z_{oe}^*)}{\left(e^{j2\beta_e l}-\rho_{1e}\rho_{2e}\right)}\pm\frac{1}{2}\frac{1}{1+Z_o/Z_{oo}}\cdot\frac{(1-Z_o/Z_{oo})e^{j2\beta_o l}+\rho_{2o}(1+Z_o/Z_{oo}^*)}{\left(e^{j2\beta_o l}-\rho_{1o}\rho_{2o}\right)} \quad (7)$$

$$S_{41,31}=\frac{1/2}{1+Z_o/Z_{oe}}\cdot\frac{(1+\rho_{2e})e^{j\beta_e l}e^{\alpha_e l/2}}{\left(e^{j2\beta_e l}-\rho_{1e}\rho_{1o}\right)}\pm\frac{1/2}{1+Z_o/Z_{oo}}\cdot\frac{(1+\rho_{2o})e^{j\beta_o l}e^{\alpha_o l/2}}{\left(e^{j2\beta_o l}-\rho_{1o}\rho_{2o}\right)} \quad (8)$$

where

$$\rho_{1e,1o}=\frac{Z_o/Z_{oe,oo}^*-1}{Z_o/Z_{oe,oo}+1}$$

and

$$\rho_{2e,2o}=\frac{Z_o e^{-\alpha_{e,o}l}/Z_{oe,oo}-1}{Z_o e^{-\alpha_{e,o}l}/Z_{oe,oo}^*+1}.$$

The remaining scattering coefficients $S_{33,43}$ and $S_{13,23}$ are obtained from (7) and (8), respectively, by changing only the sign of the taper rates $\alpha_{e,o}$ for each mode.

Throughout the previous analysis we assumed two independent taper rates for the even and odd impedances. It should be noted that physical realizability requires that [9]

$$Z_e(x)\geqslant Z_o(x), \qquad o\leqslant x\leqslant l.$$

Also for optimum coupling the condition $Z_{oe}Z_{oo}=Z_o^2$ becomes

$$Z_{oe}e^{\alpha_e x}Z_{oo}e^{\alpha_o x}=Z_o^2. \quad (9)$$

Equation (9) is satisfied by choosing $\alpha_o=-\alpha_e=\alpha$ and $Z_{oe}Z_{oo}=Z_o^2$, where Z_o is the characteristic impedance of the system or the terminating impedance at each port of the coupler. The even and odd mode impedances at any point x, measured from the tight coupling end, is then given by

$$Z_{oe}(x)=Z_{oe}e^{-\alpha x} \quad (10a)$$

$$Z_{oo}(x)=\frac{Z_o^2}{Z_{oe}}e^{\alpha x}. \quad (10b)$$

When Z_o is determined, the design parameters then become Z_{oe}, α and the length l of the coupler.

The closed-form solutions of the scattering parameters of the following special cases can be obtained from (7) and (8) as follows:

1) for a CETL directional coupler in a homogeneous medium [9], we have $\beta_e=\beta_o$;
2) for a CUTL directional coupler in an inhomogeneous medium [11] we have $\alpha e=\alpha_o=0$; and
3) for a CUTL directional coupler in a homogeneous medium, we have $\beta_e=\beta_o$ and $\alpha_e=\alpha_o=0$.

IV. Microstrip Exponential Coupler

A computer program for the analysis and optimization of these circuits has been developed. The program was used to optimize a directional coupler with a coupling coefficient of 10.5 ± 0.5 dB and with an operational band centered around 3 GHz.

The following are two ways of using the coupler.

a) With the source placed at the loose coupling side. In Fig. 2 the values of Z_{oe}, α_e, v_e, Z_{oo}, α_o, v_o, and l are given and the computed curves of the scattering parameters in decibels are also shown. The variation of S_{34} is within 3 dB of the desired coupling coefficient in the frequency range 1–7.5 GHz.

b) With the source placed at the tight coupling side. Fig. 3 shows that the reflection coefficient is improved while the coupling and isolation are still approximately as in case 1) above. The variation of S_{21} is within 3 dB of the desired coupling coefficient in the frequency range 1–10 GHz.

The values of the odd and even characteristic impedances and the taper rates are suitable for microstrip application. It should be noted from Figs. 2 and 3 that the couplings S_{21} and S_{34} have much flatter frequency characteristics compared with what is achieved using uniform lines. However, the directivity of the coupler deteriorates at high frequencies due to the rise in S_{24} and S_{31}. This is an inherent characteristic of couplers in inhomogeneous media when the phase velocities are not equal.

The practical microstrip circuit was built on a 1×1-in alumina substrate ($\epsilon_r=9.7$, $h=0.635$ mm). The relation between the physical dimensions of the microstrip coupler and the even and odd mode exponential characteristic impedances is not available. As an approximation, the equations of the uniform coupled lines are assumed to be valid at each point x along the lines. The configuration of the microstrip circuit is obtained by the following steps.

1) The length l is divided into a reasonable number of equal intervals and the values of $Z_{oe}(x)$ and $Z_{oo}(x)$ are calculated at each value of x as given in Table I and Fig. 4.

2) The corresponding values of w and s are obtained.

3) The final shape of the microstrip coupler is obtained by constructing points according to the values of x, w, and s and connecting these points by smooth curves.

The mask and the designed microstrip coupler are shown in Fig. 5. The computed and measured scattering parame-

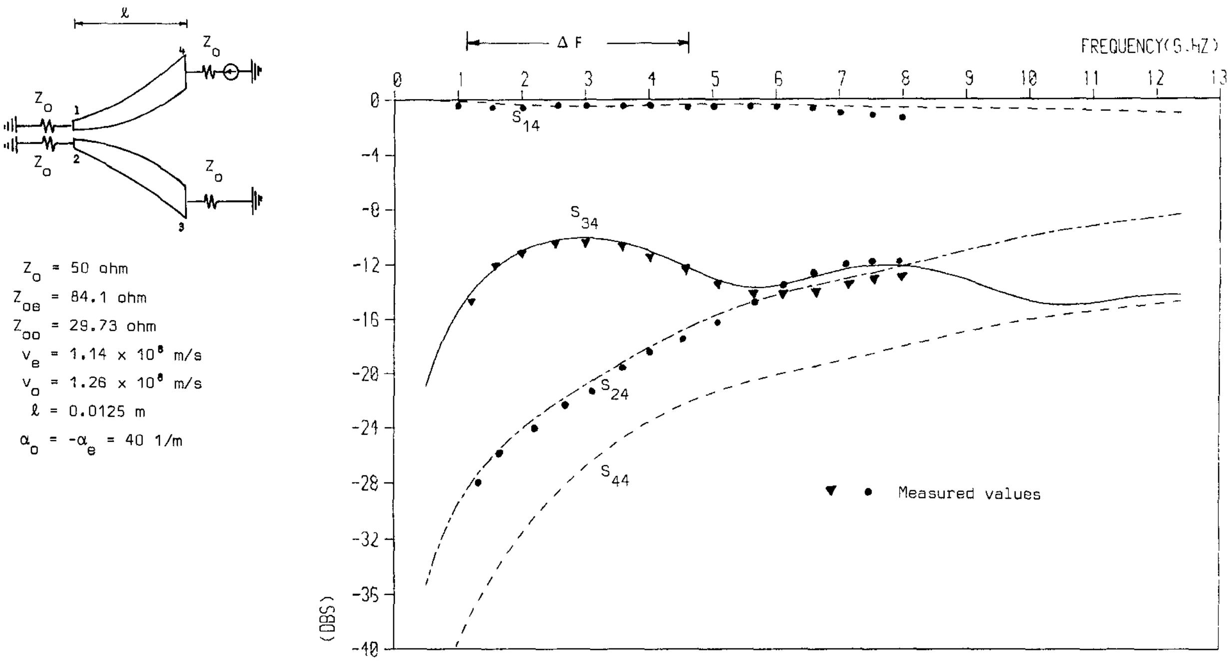

Fig. 2. The scattering parameters S_{14}, S_{24}, S_{34}, and S_{44} of a CETL directional coupler in an inhomogeneous medium.

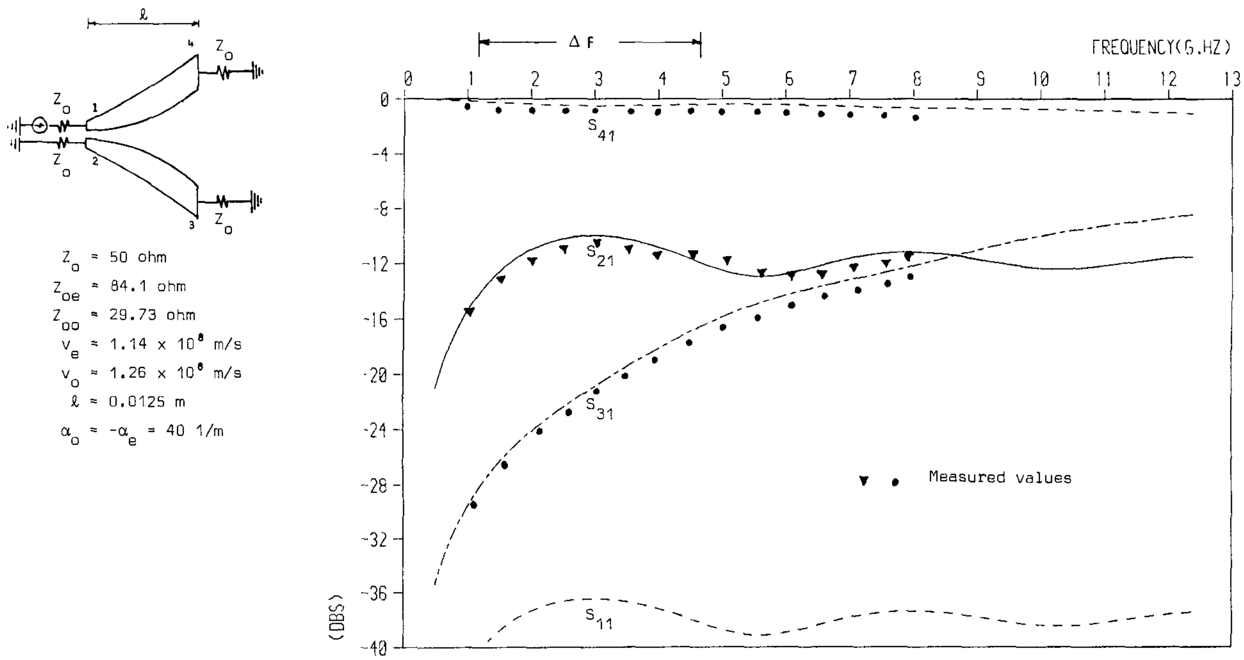

Fig. 3. The scattering parameters S_{11}, S_{21}, S_{31}, and S_{41} of a CETL directional coupler in an inhomogeneous medium.

TABLE I

x/ℓ	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$Z_{oe}(x)\Omega$	84.10	80.00	76.1	72.39	68.86	65.5	62.31	59.27	56.38	53.63	51.01
$Z_{oo}(x)\Omega$	29.73	31.25	32.85	34.54	36.31	38.17	40.13	42.18	44.35	46.62	49.01
w/h	0.655	0.725	0.78	0.835	0.885	0.92	0.99	0.995	1.01	1.04	1.055
s/h	0.065	0.09	0.135	0.195	0.28	0.4	0.58	0.85	1.27	2.15	5.0
w(mm)	0.416	0.46	0.495	0.530	0.562	0.584	0.629	0.632	0.641	0.66	0.67
s(mm)	0.041	0.057	0.086	0.124	0.178	0.254	0.368	0.54	0.807	1.365	3.175
$v_e \times 10^{-9}$ m/s	0.114	0.113	0.113	0.113	0.113	0.113	0.114	0.114	0.114	0.115	0.116
$v_o \times 10^{-9}$ m/s	0.131	0.131	0.131	0.131	0.13	0.129	0.128	0.127	0.124	0.123	0.12

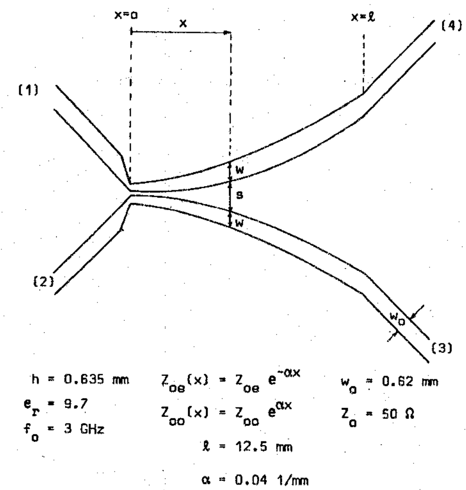

Fig. 4. The microstrip circuit of the designed coupler.

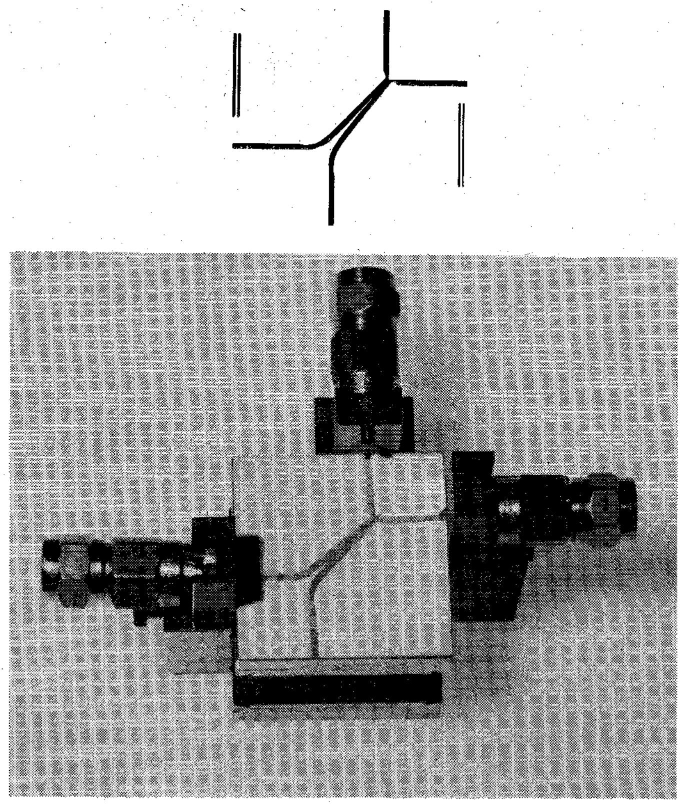

Fig. 5. Mask and photograph of the designed coupler.

ters are shown in Figs. 2 and 3. Very good agreement between theoretical and measured values was obtained.

The range of frequencies ΔF in which the coupler could be applied is marked in both Fig. 2 and Fig. 3. At high frequencies the directivity of the coupler deteriorates due to the rise in S_{24} and S_{31} which is caused by the unequal phase velocities v_e and v_o.

V. Conclusion

The solution derived for the characterization of CETL in inhomogeneous media gives accurate design parameters for microwave couplers. Microstrip couplers using CETL have a much flatter frequency response than those using uniform lines. Due to unequal phase velocities, the directivity of the coupler deteriorates at high frequencies. If methods of equalizing the mode velocities are employed, then couplers with flat characteristics and good directivities over several octaves could be designed.

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Suspended Broadside-Coupled Slot Line with Overlay

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***Abstract*—This paper presents a rigorous analysis of symmetric, broadside-coupled slot line with overlay. The structure is assumed to be suspended inside a conducting enclosure of arbitrary dimensions. The dielectric substrate and the overlay are assumed to be isotropic and homogeneous and are of arbitrary thickness and relative permittivity. The conducting enclosure and the zero thickness metallization on the substrate are assumed to have infinite conductivity. The computed results illustrate a) the dispersion characteristics and characteristic impedance of the coupled slot line structure, b) the variation of the even-mode and also the odd-mode relative wavelength ratio and characteristic impedance with slot width, and c) the effect of shielding on the even-mode and also the odd-mode dispersion and characteristic impedance. This structure should find application in the design and fabrication of MIC components such as magic-T's and directional couplers.**

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I. Introduction

THE PARALLEL-COUPLED slot line on a dielectric substrate is ideally suited for microwave integrated circuit components such as Magic-T's [1] and directional couplers [2], [3]. Recently, parallel-coupled slot line on double layer dielectric substrate and parallel-coupled slot line sandwiched between two dielectric substrates have been reported [4].

The paper presents an analysis of symmetric, broadside-coupled slot line with overlay and suspended inside a conducting enclosure of arbitrary dimensions. The dielectric substrate and the overlay are assumed to be isotropic